A FUZZY LOGIC ALGORITHM FOR OPTIMAL ALLOCATION OF DISTRIBUTED RESOURCES

Dr. James F. Smith III
Naval Research Laboratory, Code 5741
Washington, D.C., 20375-5000
Telephone: 202.767.5358
Fax: 202.404.7690
jfsmith@drsews.nrl.navy.mil

Robert D. Rhyne II
Naval Research Laboratory, Code 5741
Washington, D.C., 20375-5000
Telephone: 202.767.5938
Fax: 202.404.7690
rrhyne@drsews.nrl.navy.mil

ABSTRACT- *A fuzzy logic based resource manager (RM) that will allocate resources distributed across many platforms is under development. The platforms will consist of ships, aircraft, etc. The resources will be various sensors: ESM, RADAR, IFF, and communications. The RM will allow codification of military expertise in a simple mathematical formalism known as the fuzzy decision tree. The fuzzy decision tree will form what is known as a fuzzy linguistic description, i.e., a formal fuzzy if-then rule based representation of the system. Since the decision tree is fuzzy the uncertainty inherent in the root concepts propagates throughout the tree. The functional form of the fuzzy membership functions for the root concepts will be selected heuristically and will generally carry one or more free parameters. The free parameters in the root concepts will be determined by optimization both initially and later at non-critical times. A genetic algorithm will be used for optimization.*

Keywords: fuzzy logic, genetic algorithms, expert systems, multisensor data fusion, distributed AI algorithms

1. Introduction

Modern naval battleforces generally include many different platforms each with its own sensors, radar, ESM, and communications. The sharing of information measured by local sensors via communication links across the battlegroup should allow for optimal or near optimal decisions. The survival of the battlegroup or members of the group depends on the automatic real-time allocation of various resources.

A fuzzy logic algorithm has been developed that automatically allocates electronic attack (EA) resources in real-time. The particular approach to fuzzy logic that will be used is the fuzzy decision tree, a generalization of the standard artificial intelligence technique of decision trees [1].

The controller must be able to make decisions based on rules provided by experts. The fuzzy logic approach allows the direct codification of expertise forming a fuzzy linguistic description [2], i.e., a formal representation of the system in terms of fuzzy if-then rules. This will prove to be a flexible structure that can be extended or otherwise altered as doctrine sets, i.e., the expert rule sets change.

The fuzzy linguistic description will build composite concepts from simple logical building blocks known as root concepts through various logical connectives: “not”, “and”, “or”, etc. Optimization will be conducted to determine the form of the membership functions for the fuzzy root concepts.

Section 2 gives a brief introduction to the ideas of fuzzy set theory, fuzzy logic, decision trees, root and composite concepts. Section 2 uses these concepts to develop the kinematic-ID subtree, which is an important component of the decision tree. Section 3 describes the optimization of the resource manager’s performance. Section 4 provides an example of the algorithm’s allocation of EA resources distributed over three platforms against an airborne targeting radar with uncertain ID. Section 5 discusses association algorithms and points out the usefulness of a particular fuzzy logic based association algorithm. Section 6 discusses future developments. Finally, section 7 provides conclusions.

2. A Brief Introduction to Fuzzy Sets, Logic, and Decision Trees

The resource manager (RM) must be able to deal with linguistically imprecise information provided by an expert. Also, the RM must control a number of assets and be flexible enough to rapidly adapt to change. The above requirements suggest an approach based on fuzzy logic. Fuzzy logic is a mathematical formalism that attempts to imitate the way humans make decisions. Through the concept of the grade of membership, fuzzy set theory and fuzzy logic allow a simple mathematical expression of uncertainty. The RM will require a mathematical representation of domain expertise. The decision tree of classical artificial intelligence provides a graphical representation of expertise that is easily adapted by adding or pruning limbs. Finally, the fuzzy decision tree, a fuzzy logic extension of this concept, allows easy incorporation of uncertainty as well as a graphical codification of expertise.

This section will develop the basic concepts of fuzzy sets, fuzzy logic, and fuzzy decision trees. Examples from a primitive military doctrine set will be provided.

2.1 Fuzzy Set Theory

This subsection provides a basic introduction to the ideas of fuzzy set theory. Fuzzy set theory allows an object to have partial membership in more than one set. It does this through the introduction of a function known as the membership function, which maps from the complete set of objects *X* into a set known as membership space. More formally, the definition of a fuzzy set [3] is

If *X* is a collection of objects denoted generically by *x* then a fuzzy set *A* in *X* is a set of ordered pairs:

$$A = \{(x, m_A(x)) | x \in X\}$$

$m_A(x)$ is called the membership function or grade of membership (also degree of compatibility or degree of truth) of *x* in *A* which maps *X* to the membership space *M.*

The logical connectives "and", "or", and "not" are defined as

$$or: A \cup B \rightarrow m_{A \cup B}(x) = \max[m_A(x), m_B(x)]$$
$$and: A \cap B \rightarrow m_{A \cap B}(x) = \min[m_A(x), m_B(x)]$$
$$not \quad B: \overline{B} \rightarrow m_{\overline{B}}(x) = 1 - m_B(x)$$

2.2 Fuzzy Decision Trees and Root Concepts

In this section methods of constructing classical and fuzzy decision trees are discussed. The fuzzy decision tree will provide a graphically intuitive way of propagating information from basic to complex concepts.

A classical decision tree is a standard artificial intelligence technique for making decisions. It's graphical nature allows an easy intuitive representation of information. The method of constructing decision trees, both classical and fuzzy, is best illustrated through an example. Consider the following simple military doctrine set, i.e., a set of rules provided by an expert:

R1: IF target is *Attacking* or *Bearing-in* or *Maneuvering*, THEN the target is *Important*
R2: IF target is *Close* and not *Friend*, THEN the target is *Attacking*.

These rules can be represented in a tree form which is given in Figure 1.

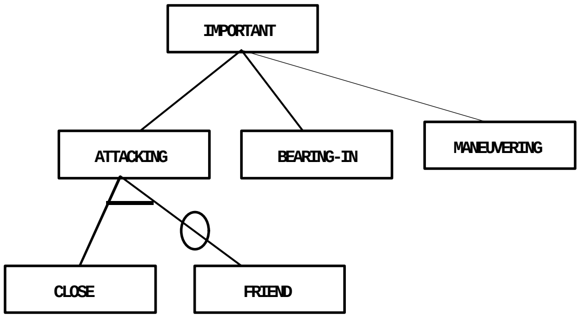

Figure 1 : Decision Tree for rules R1 and R2

In Figure 1 the root concepts are "close", "friend", "bearing-in", and "maneuvering". The composite concepts are "attacking" and "important". The root and composite concepts are placed in their own boxes. The boxes are connected with lines. Vertices marked with a horizontal line are read as "and", unmarked vertices as "or", and lines marked by a circle indicate negation.

The conversion from a classical decision tree to a fuzzy decision tree is carried out by

-assigning each classical root concept, those boxes at the bottom-most level of the decision tree, membership functions and then

-converting all classical "or", "and", and "not" operations to the analogous fuzzy operations.

So for track *i,* the following grades of membership associated with the corresponding root concepts must be defined:

$$m_{friend}, m_{close}, m_{bearing-in}, \text{and } m_{maneuvering}$$

Pursuing the second component of the above description, i.e., the conversion of classical "and", "or", and "not" into the related fuzzy set-theoretic quantities, gives the following grades of membership for the composite concepts "attacking" and "important":

$$m_{attacking}(i) = \min[m_{close}(i), 1 - m_{friend}(i)]$$

$$m_{\text{important}}(i) =$$
$$\max[m_{attacking}(i), m_{bearing-in}(i), m_{maneuvering}(i)]$$

$$m_{\text{important}}(i) =$$
$$\max[\min[m_{close}(i), 1 - m_{friend}(i)],$$
$$m_{bearing-in}(i), m_{maneuvering}(i)]$$

The resulting grades of membership for composite concepts are used for establishing priorities for resource allocation.

Figure 1 is referred to as the kinematic-ID subtree. It is a subtree of a larger fuzzy decision tree used by an isolated ship for allocation of its own EA resources. Each ship in the battlegroup has an isolated ship tree that allocates its EA resources. These isolated ship trees, when linked together by information from line of sight communication form a larger tree, known as the multi-platform tree. It is this tree together with information sent over communications links, that determines allocation of EA resources over the entire battlegroup. The full isolated ship tree, communication models, and the multi-platform tree will not be discussed in detail here due to space limitations. A more detailed account of these concepts will be published in the near future [4].

2.3 Root Concept Membership Functions

The next step required for implementation of the fuzzy linguistic description is defining membership functions for the root concepts. There is not an a priori best membership function so a reasonable mathematical form is selected. This subjective membership function will be given in terms of one or more parameters that must be determined. The parameters may be set initially by an expert or they may be the result of the application of an optimization algorithm. The possible use of a stochastic optimization algorithm to determine the unknown parameters in root concept membership functions is discussed in section 3.

As a first example of a membership function definition consider the root concept "close." The concept "close" refers to how close the target/emitter on track *i* is to the ship, or more generally platform of interest. The universe of discourse will be the set of all possible tracks. Each track *i* has membership in the fuzzy set "close" based on its range R (nmi) and range rate *dR/dt* (ft/sec). An appropriate membership function might be

$$m_{close}(i) = \frac{1}{1 - a\,|R_i - R_{\min}|\,/\max(-\dot{R}_i, \dot{R}_{\min})}$$

The parameters to be determined by optimization are

$$a, \quad R_{\min}, \quad \text{and} \quad \dot{R}_{\min}.$$

3. Optimization

There are many different types of optimization algorithms found in the literature. Many of these algorithms are known as greedy algorithms because they will find as a solution the first extremum encountered in a parameter space. Examples of this kind of algorithm are found in reference [5].

An algorithm that has the capability to explore parameter space before settling on a solution, intuitively would seem to have greater probability of selecting an optimal or near-optimal solution than a greedy algorithm. Examples of algorithms of this kind are stochastic optimization algorithms, which include simulated annealing [5] and genetic algorithms [6].

A genetic algorithm (GA) is an optimization method that manipulates a string of numbers in a manner similar to how chromosomes are changed in biological evolution. An initial population made up of strings of numbers is chosen at random or is specified by the user. Each string of numbers is called a "chromosome" or an "individual," where each number slot is referred to as a "gene." A set of chromosomes forms a population where each chromosome represents a given number of traits that are the actual parameters being varied to optimize the "fitness function". The fitness function is a performance index that we seek to maximize.

The operation of the genetic algorithm proceeds in steps. Beginning with the initial population, "selection" is used to choose which chromosomes should survive to form a "mating pool." Chromosomes are chosen based on how "fit" they are (as computed by the fitness function) relative to the other members of the population. More fit individuals retain more copies of themselves in the mating pool so that they will have greater representation in the next generation. Next, two operations are taken on the mating pool. First, "crossover" (which represents mating, the exchange of genetic material) occurs between parents.

In crossover, a random spot is picked in the chromosome, and the genes after this spot are switched with the corresponding genes of the other parent. Following this, "mutation" occurs. Mutation represents the change of values of randomly selected genes in a chromosome. After the crossover and mutation operations occur, the resulting strings form the next generation and the process is repeated. A termination criterion is used to specify when the genetic algorithm should end (e.g., the maximum number of generations or until the maximum fitness exhibits little or no change over a certain number of generations).

The following characteristics are also considered advantages of the genetic algorithm:

- the genetic algorithm works on a population of points, not a single point,
- they work directly with strings of characters representing the entire parameter set, not the individual parameters,
- the search is guided by probabilistic rules, not deterministic rules. The inherent randomness in this procedure allows the genetic algorithm to escape local maxima,
- genetic algorithms, like simulated annealing represent a form of optimization that does not require derivatives. The genetic algorithm only requires information about how fit a given solution is, i.e., the effect of the solution on the fitness function.

The construction of good fitness functions for this application requires insight in four areas, with the rules being derived from geometry, physics, engineering, and military doctrine. Several classes of fitness functions are being explored. The fitness functions tend to be highly nonlinear and non-differentiable at many points. For classical optimization algorithms, the non-differentiability might have posed a problem, but it offers no difficulty for a genetic algorithm.

The fitness functions currently being explored are expressible mathematically as a linear combination of products of Heaviside step-functions [7]. The step function arises from the rule-based origin of the fitness functions. The arguments of the fitness functions are given by the difference of the membership function and a parameter characteristic of expertise. The linear combinations of products of the step functions are typically averaged over an ensemble of kinematic scenarios, where each element of the ensemble differs from the others in terms of initial conditions. For example, the ensemble used to optimize the membership function for the root concept "close" consists of elements with different initial values for range, and its first two derivatives with respect to time. From these initial values, the range and range rate are calculated as a function of time allowing the membership function for "close" to be optimized over many physical scenarios. This is referred to as a geometric-kinematic ensemble. Despite the complicated non-linear form that the fitness function takes because of the rules used in its construction, genetic algorithm based optimization has proven to be effective.

The method described above for constructing fitness functions is only a first step. The fitness functions constructed in this manner, are most applicable to isolated platforms. The ultimate goal is to construct a resource manager/scheduler that is optimal in its performance when dealing with multiple dissimilar platforms. By pursuing the isolated platform model first, the region of parameter space that must be explored for the multi-platform problem is reduced. It would be expected, on intuitive grounds, that parameters for the multi-platform problems should lie within some neighborhood, of those solutions for the isolated platform model. The motivation for this assumption is that at any given time, each platform may be called upon to defend itself. Once the isolated platform parameters are selected for each root concept membership function, neighborhoods around these parameters can be defined, and a parameter space for the multi-platform problem formed by constructing a product space from the coordinate spaces defined by each isolated platform neighborhood. Therefore, the potentially large parameter space that must be explored for the multi-platform problem is constrained through the use of a priori information, significantly reducing the run-time of the genetic algorithm. This procedure has proven effective in producing very high quality multi-platform performance. The performance of the model and the potential risk of restricting parameter space in this way will be examined in a future paper [4].

4. An Example of Multi-platform Response

In this section a specific example of the fuzzy RM's ability to optimally allocate electronic attack resources is examined. Input requirements and output characteristics are considered, and illustrated through the actual output of the current implementation of the RM.

4.1 Input Scenario

The fuzzy RM requires as input, the position and number of ally platforms, e.g., ships, planes, etc., also emitter range, bearing, heading, elevation, and an emitter ID with an uncertainty associated with the ID. The effect of the data is to stimulate the various kinematic concepts like "close" resulting in different "actions" by the algorithm. The emitter ID is used to determine the technique or techniques (for ID's with uncertainty) that the ally platform or platforms can execute against the emitter.

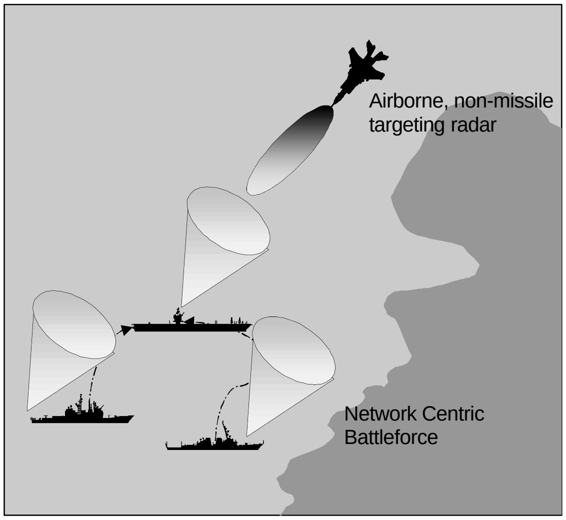

Figure 2: The fuzzy RM allocates EA resources distributed over three ships against a targeting radar with uncertain ID.

In Figure 2, there is a battleforce of three ships and also an incoming aircraft with targeting radar. However, in this scenario, the type of the threat emitter is not well-known. With the threat's classification not being well-known, and because the uncertainties indicate a foe of some type, all three ships conduct joint EA against the threat emitter.

The ship acting as command ship sends communication over the network to other adjacent ships asking for joint EA and chooses the electronic counter measures (ECM) technique most likely to be effective against this type of threat. The adjacent ships choose two other ECM techniques based on the emitter's ID and its uncertainty.

It should be noted, each ship has the same software aboard, and can act as a command ship. This significantly reduces the likelihood of the battlegroup being rendered ineffective by the loss of a single platform.

4.2 Output of the Fuzzy RM

In Figure 3, the algorithm's output for the scenario in Figure 2 is displayed. A polar plot with origin at the centroid of battlegroup is used to display the positions of the three ships (diamonds), the incoming emitter (triangle marked with designation "foe type"), and friendly aircraft (triangles marked with the designation "friend type"). Communications and electronic attack techniques used by each ship are listed to the side. The arrows running from the ships to the foe-type emitter indicate electronic attack.

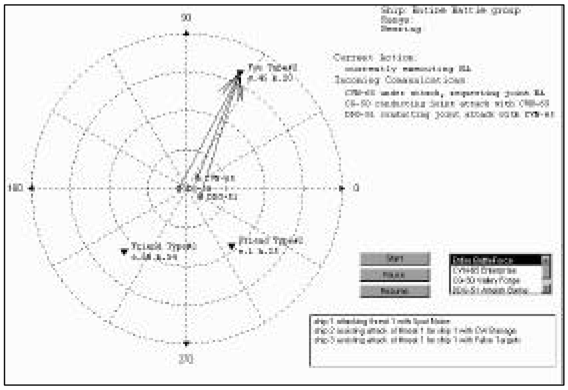

Figure 3: The algorithm's output showing how it allocates EA resources distributed over three platforms.

The algorithm, during its real-time run, displays an image of this type every second. As indicated in the box in the right-hand corner of Figure 3, the algorithm chooses the appropriate techniques for all three attacking ships. As consistent with military doctrine, all three ships are conducting joint EA. Finally, it should be noted there are two friendly aircraft in the scenario. The algorithm will not attack an emitter based on kinematic properties if the emitter has been clearly identified as a friend.

The algorithm has been determined to be effective by comparing its output to the judgement of human experts. Statistical evaluation of the algorithm's effectiveness will be published in the near future [4].

5. Dealing with Imperfect Association

It is assumed that the data provided as input to the RM has already been associated, i.e., the appropriate ESM and radar data have already been perfectly assigned to the same emitter. Association of the ESM and radar data is valuable since radar provides range and bearing information for use in the root concept "close" and ESM can provide ID, bearing, RF and PRI of the emitter. Unfortunately, the association of ESM and radar is generally not perfect given the sparse, intermittent and noisy nature of data.

The abilities of two different association algorithms to associate data as a function of the measured ESM points will be compared. These algorithms are the fuzzy association algorithm described in reference [8-12] and a Bayesian philosophy algorithm described in reference [13] and referred to here as the TW-algorithm.

The two association algorithms are compared using the same simulated ESM and radar data. The emitter has a bearing of 0 degrees. This is absolute truth for this simulation. Radar has determined there are objects traveling with bearings of 0, 1, and –1 degrees. For simulation purposes zero mean Gaussian noise with 1 degree standard deviation is added to simulate noise in the ESM measurement process. This is a difficult association problem since there are radar measurements not only at 0 degrees, but also radar measurements within one standard deviation of truth.

Since the radar measurements contain truth it is expected that a good association algorithm will associate the zero degree radar track with the ESM data. A probability of association between each radar track and the ESM data is calculated as in references [8-13]. Both algorithms give rise to five hypothesis classes describing whether or not the ESM data is associated with a radar track. It is desirable that when radar contains "truth," i.e., in this case the zero degree track, the track corresponding to truth, be firmly correlated with the ESM data. In this way the probability of making an inappropriate assignment of range is minimized. The notion of firm correlation is defined in detail in the references [8-13]. The other hypothesis classes will not be displayed, as they are not interesting for the example that follows and only serve to obscure the results.

Both the fuzzy association and TW-algorithms can be used to associate noisy ESM and noisy radar measurements [8-12]. The radar measurements for radar track *j* at time t_i will have zero mean Gaussian noise added to them. The variance of the noise will be denoted as s_{ij}^2 for the j^{th} radar track at the i^{th} time.

Figure 4 presents results for three radar tracks with the following bearings: $m = 0^o, \ 1^o, \ -1^o$ with $s_{ij} = 0.1^o$ for all times t_i and radar tracks *j*. The radar noise standard deviation is consistent with levels found in modern radar systems. Since the radar results contain truth, i.e., a target moving with constant bearing of 0^o a good association algorithm will establish that there is a firm correlation between the ESM data and the 0^o bearing track. Figure 4 plots the probability the association algorithms establish a firm association between ESM data and the radar measurements. The fuzzy association algorithm results are given by the curve marked with *o*'s and the TW results are indicated by the curve marked with +'s. The vertical axis indicates probability of firm correlation and the horizontal axis the number of data points necessary to establish that level of probability.

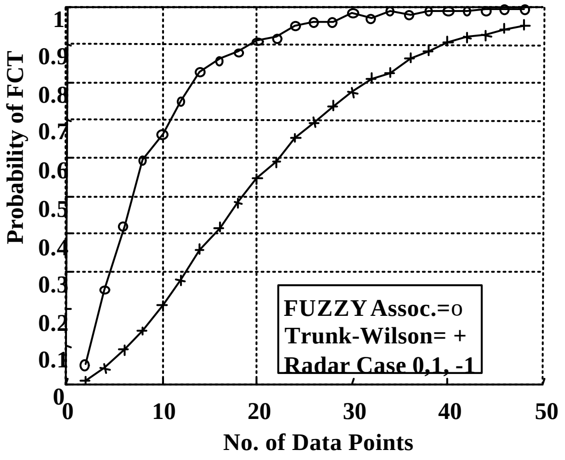

Figure 4: Fuzzy and Bayesian association

The fuzzy association algorithm results are always superior to the TW-algorithm. At ten data points the fuzzy algorithm has established a 65% probability of firm correlation, between the ESM data and the 0^o radar track. The TW-algorithm requires about 24 points to establish the same level of probability of FCT. The fuzzy algorithm establishes an 80% probability of FCT by the 12th data point, whereas the TW-algorithm requires about 30 points to reach the same level of success. The fuzzy algorithm reaches 90% probability of FCT at 20 data points and the TW-algorithm at about the 38th point. Therefore, the fuzzy algorithm establishes high probabilities of firm correlation with between 1/3 to 1/2 the data required by the TW-algorithm. In this sense the fuzzy algorithm is 2 to 3 times faster than the TW-algorithm. Also, this is a difficult example for any association algorithm since there are two additional radar measurements within one noise standard deviation. The results are only slightly inferior to the case where radar is simulated as noiseless as found in reference [11]. The ability of the fuzzy algorithm to make high quality decisions with much less data than the TW-algorithm is significant since real data is frequently sparse and intermittent.

The above examples are for the case where there is 100% detection of ESM and radar data. In reference [11] it is shown with a detection rate as low as 70% of the ESM points, the fuzzy association algorithm experiences little deterioration, whereas the TW-algorithm's performance is greatly degraded.

The example in Figure 4 is for the case of a single emitter. In reference [11] it is shown that the fuzzy association algorithm gives a similar level of performance if there are one, four or 10 emitters, even when ESM detection rates drop down to 70%. In particular, for 10 emitters closely spaced in the RF-PRI plane the fuzzy association algorithm displays results like those found in Figure 4, but the TW-algorithm deteriorates more than 40% by the 48th data point.

The use of the fuzzy association algorithm will allow association decisions to be made with 1/6 to ½ the data required by the Bayesian association algorithm. Faster association of ESM and radar tracks means better assignment of range and ID's to potential threats. As a final observation, the use of both a fuzzy RM and a fuzzy association algorithm would allow linguistic data to be shared between the two algorithms. This should increase the effectiveness of both algorithms. The easy sharing of linguistic rules and other linguistic data is not an option, if a non-fuzzy association algorithm like the TW algorithm were to be used for association.

6. Future Developments

There are several activities that will be conducted in the near future, which include: expansion of the rule set, research related to improved optimization, expansion of the technique library, the invention of new multi-platform electronic attack techniques which make good use of the resources distributed over multiple platforms, and validation of the multi-platform resource manager.

7. Conclusions

A fuzzy logic based algorithm for optimal allocation and scheduling of electronic attack resources distributed over many platforms is under development. The kinematic-ID subtree that forms the core of the isolated ship model has been discussed and used to illustrate the mathematical concepts involved. Root concept membership function construction has been discussed. Optimal performance for the algorithm is obtained by selecting values of the free parameters in the root concept membership function using a genetic algorithm. The use of a genetic algorithm requires the construction of a fitness function. The fitness functions constructed for this task are based on insights obtained from geometry, physics, engineering, and military doctrine. The fitness functions are in general non-differentiable and highly non-linear, neither property providing an obstacle for a genetic algorithm. Finally, fuzzy logic based multi-sensor association should prove very effective both in its ability to form high quality conclusions faster than a standard Bayesian algorithm and because it allows linguistic data to be shared easily between the resource manager and the multi-sensor association algorithm.

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9. References

[1] J.M. Molina Lopez, F.J. Jimenez Rodriguez, and J.R. Casar Corredera, "Symbolic Processing for Coordinated Task Management in Multiradar Surveillance Networks," *Fusion98, Proceedings of the International Conference on Multisource-Multisensor Information Fusion*, R. Hamid, and Dongping Zhu, Vol . II, pp. 725-732, CSREA Press, Las Vegas, Nevada, USA, July 6-9,1998.

[2]L.H. Tsoukalas and R.E. Uhrig, *Fuzzy and Neural Approaches in Engineering*, (John Wiley and Sons, New York, 1997).

[3]H. J. Zimmerman, *Fuzzy Set Theory and its Applications*, page 11,(Kluwer Academic Publishers Group), Boston, 1991.
[4] J.F. Smith, III and R. Rhyne, II, “A General Fuzzy Logic Resource Manager and Communication Issues for Distributed Resources,” to be published, 2000.
[5] W.H. Press, S.A. Teukolsky, W.T. Vetterling, and B.P. Flannery, *Numerical Recipes in Fortran: The Art of Scientific Computing*, (Cambridge University Press, 1992).
[6] D.E. Goldberg, *Genetic Algorithms in Search, Optimization and Machine Learning* (Addison-Wesley, Reading, MA, 1989).
[7] A.I. Saichev and W.A. Woyczynski, *Distributions in the Physical and Engineering Sciences*, (Birkhauser, 1997).
[8] J. F. Smith III, “A Fuzzy Logic Multisensor Association Algorithm: Theory and Simulation,” Naval Research Laboratory Technical Formal NRL/FR/5740--97-9866, Naval Research Laboratory, Washington D.C., 20375-5000, 9/97
[9] J. F. Smith III, “A Fuzzy Logic Multisensor Association Algorithm,” *Signal Processing, Sensor Fusion, and Target Recognition VI*, I. Kadar, Vol. 3068, pp. 76-87, SPIE Proceedings, Orlando, FL, April 21-24, 1997
[10] J. F. Smith III, “A Fuzzy Logic Multisensor Association Algorithm Dealing with Multiple Targets Intermittent Data and Noise” , *Signal Processing, Sensor Fusion, and Target Recognition VII*, I. Kadar, Vol. 3374, pp. 2-13, SPIE Proceedings, Orlando, FL, April 13-17, 1998.
[11] J.F. Smith III, “A Fuzzy Logic Multisensor Association Algorithm: Applied to Noisy, Intermittent and Sparse Data”, Fusion98, Proceedings of the International Conference on Multisource-Multisensor Information Fusion, , R. Hamid, and Dongping Zhu, Vol . II, pp. 681-687, CSREA Press, Las Vegas, Nevada, USA, July 6-9,1998.
[12] J. F. Smith III, “A Fuzzy Logic Multisensor Association Algorithm: Multiple Emitters, Computational Complexity, and Noisy Data”, Naval Research Laboratory Technical Formal Report, Naval Research Laboratory, Washington D.C., 20375-5000, to be published 1999.
[13] G.V. Trunk and J.D. Wilson, “Association of DF Bearing Measurements with Radar Track,” IEEE Trans on AES, Vol. AES-23., No. 4, July 1987