A Fuzzy Logic Multisensor Association Algorithm: Applied to Noisy, Intermittent and Sparse Data

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ABSTRACT

A recursive multisensor association algorithm has been developed based on fuzzy logic. It simultaneously determines fuzzy grades of membership and fuzzy cluster centers. It is capable of associating data from various sensor types and in its simplest form makes no assumption about noise statistics as many association algorithms do. The algorithm is capable of performing without operator intervention. It associates data from the same target for multiple sensor types. The algorithm also provides an estimate of the number of targets present, reduced noise estimates of the quantities being measured, and a measure of confidence to assign to the data association. A comparison of the algorithm to a more conventional Bayesian association algorithm is provided. The data for both the ESM and radar systems is noisy. Examples are included in which the data has probability of detection less than unity.

Keywords: fuzzy clustering, fuzzy sets, fuzzy logic, recursive, multisensor, association, superclustering, outlier suppression, ESM, radar

1. Introduction

The problem considered in this paper is how to associate Electronic Support Measures (ESM) signals with one or more of m possible radar tracks. An algorithm based on fuzzy set theory has been developed to solve this problem [1]. It considers the complexities offered by having multiple radar tracks and unequal numbers of measurements. It is capable of making its own estimate from ESM data of bearing and as such provides additional measures of association unlike the Trunk-Wilson (TW) Bayesian theory [2] with which it is compared. It can estimate the number of targets present in the data and use fuzzy set theoretic techniques to suppress outliers. The fuzzy grades of membership provide opportunities for incorporation of heuristic rule sets and extension to probability theory. The fuzzy cluster centers represent reduced noise estimates of the measured quantities. Finally, in comparison to an existing Bayesian algorithm the fuzzy association algorithm exhibits superior performance.

In section 2, the concepts of fuzzy set theory, hard and fuzzy clustering, and superclustering are introduced. Section 3 discusses the application of the fuzzy association algorithm to simulated ESM and radar data. In this section results from the fuzzy and TW-association algorithms are compared. In section 3 noisy radar measurements are considered in different examples and compared to the results for similar noiseless radar calculations previously published [1]. The effects of random data loss and also multiple targets present in the data are examined. Section 4 considers the effect that introducing sliding windows has on the performance of the fuzzy clustering and superclustering algorithms as well as their CPU time requirements. Finally, section 5 provides conclusions.

2. Fuzzy Clustering and Superclustering

The development of the fuzzy association algorithm requires the concepts of the fuzzy set, clustering, fuzzy clustering, and superclustering. These concepts are developed in the following subsections.

2.1 Fuzzy Clustering

Fundamental to the development of the fuzzy association algorithm is the concept of clustering. Clustering is an operation that allows data to be grouped into classes defined by a similarity measure. By definition [3], given K objects the algorithm forms N clusters such that with respect to the similarity measure, the members of each cluster have a greater similarity to each other, than to the members of any other cluster.

The kind of clustering used here for association is known as fuzzy clustering. A batch version of the algorithm [4] has been previously described elsewhere.

In the development of the fuzzy association algorithm clustering will play a significant role. The grades of membership will be established by minimizing a functional. This functional can be found in many places in the literature of fuzzy sets and fuzzy clustering [5,6] and is given below:

$$J_m(U,\mathbf{v}) = \sum_{k=1}^{n}\sum_{i=1}^{c}(u_{ik})^m (d_{ik})^2$$

where c= the number of clusters, n= the number of data points, x_k = the kth data vector, u_{ij}= fuzzy grade of membership of the kth data point in the ith fuzzy cluster, v_i is the ith fuzzy cluster center, and

$$(d_{ik})^2 = \|x_k - v_i\|^2 \text{ and } \|\cdot\|$$

is any inner product induced norm on R^p. An effective value for the quantity m is 2 as discussed in references [1,4,5].

The goal of the fuzzy clustering algorithm is to determine fuzzy cluster centers v_i that represent the average value of the quantities in the fuzzy clusters, and the grade of membership of the kth data point in the ith fuzzy cluster for all data points-k and clusters-i. The algorithm determines these quantities by minimizing a least square cost function where each term is weighted by a power of the grade of membership. Each term of the cost function simultaneously measures the distance of the data point from a cluster center and is weighted by the point's membership in that cluster. The minimization is conducted subject to the constraints that the sum of the grades of membership over clusters for a particular data point must equal unity, and for each cluster the sum of grades of membership over data points must be bound between 1 and the maximum number of data points. This is referred to as a fuzzy c-means algorithm [6].

2.2 Superclustering

Clustering algorithms, including the fuzzy clustering algorithm generally require a specification of the final number of clusters. If the data being clustered represents ships, aircraft, missiles, etc., this implies *a priori* knowledge of the number of targets. Obviously, the number of targets will not be known in general before processing. So it is desirable to develop a technique for determining from the data the appropriate number of clusters, i.e., the number of targets. Such a technique, known as *superclustering*, has been developed which provides a solution to this problem. The superclustering techniques developed here are related to and represent an extension of techniques in fuzzy cluster validity theory [6].

The method of superclustering is described in greater detail elsewhere [1,4] and summarized here. The fuzzy clustering algorithm produces a pre-assigned number of clusters for the data with associated grades of membership for each data point in each cluster. The fuzzy clustering algorithm also provides the coordinates of the fuzzy cluster centers. Intuitively, clusters should be separated; non-overlapping, and not extremely close to each other with respect to some measure. It then becomes essential to define a measure of "closeness" and provide a criterion for what "too close" means.

One such normalized measure of cluster center separation is the *c-matrix* defined below. Let $v(i)$ and $v(j)$ be the position vectors for the fuzzy cluster centers for cluster i and cluster j, respectively, and n the number of data points. Then the $i^{th} - j^{th}$ element of the c-matrix is

$$c(i,j) = \| v(i) - v(j) \| / \max(std(i), std(j)) \quad (1)$$

where

$$std(k) = \sqrt{\sum_{i=1}^{n} u(i,k)^m * (x(i) - mean(k))^2 / \sum_{i=1}^{n} u(i,k)^m} \quad (2)$$

and

$$mean(k) = (\sum_{i=1}^{n} u(i,k)^m * x(i)) / \sum_{i=1}^{n} u(i,k)^m \quad (3)$$

Equations (2) and (3) define the fuzzy standard deviation and the fuzzy mean, respectively.

The *c-matrix* capitalizes on the intuitive idea that cluster centers should be separated by a certain number of fuzzy standard deviations. If cluster centers are closer than this, they probably correspond to the same cluster. If it is determined that two or more clusters should be merged into a single cluster, the resulting grouping will be referred to as a *supercluster*. A criterion must be established to determine when supercluster formation is warranted. A simple criterion consists of defining a threshold τ, such that if $c(i,j) < \tau$ then clusters i and j are merged into a supercluster. Methods for selecting the value of τ are discussed in references [1,4].

3. Application of the Fuzzy Association Algorithm to Simulated Data and Comparison to the TW-Algorithm

In this section the fuzzy association algorithm is examined for two different kinds of test cases and compared to the TW-algorithm. In subsection 3.1 there is one target with constant bearing of 0°. The ESM measurements are simulated by adding 1° standard deviation Gaussian noise to the true bearing 0°. There are two radar cases each with three noisy radar measurements. All ESM and radar points are detected. Gaussian noise with 0.1 standard deviation is added to the radar measurements. In subsection 3.2 the radar is noisy as before, but the additional feature is added that not all ESM and radar points are detected. Finally, in subsection 3.3 the fuzzy and TW association algorithms are applied to an example where there are 10 emitters closely spaced in the RF and PRI plane.

3.1 Association of Noisy ESM and Noisy Radar Measurements

In a previous paper [1], the fuzzy association and TW-association algorithms were compared for noiseless radar. The TW-algorithm can be used to associate noisy ESM and noisy radar measurements [7]. The notions of the ESM and a radar track being firmly correlated (FCT) or firmly not correlated (FNT) as defined in references [1, 2] will be used throughout. The other hypothesis classes will not be displayed as they are not interesting for the examples that follow and only serve to obscure the results. The radar measurements for radar track j at time t_i will have zero mean Gaussian noise added to them. The variance of the noise will be denoted as σ_{ij}^2 for the j^{th} radar track and the i^{th} time.

In this section two simulation classes are considered. In each case the target has a constant bearing of 0 °, and there are three constant bearing radar estimates. The radar estimates are what provide the distinction between the two examples and are given below:

Example 1 $\mu_1 = 0^o$, $\mu_2 = 1^o$, $\mu_3 = -1^o$;

Example 2 $\mu_1 = 2^o$, $\mu_2 = 1^o$, $\mu_3 = -1^o$.

In each simulation 2 data points are added each time until total of 50 data points are accumulated. In all the simulations, ensemble averages have been conducted.

Figure 1 presents results for the radar example $\mu = 0^o$, 1^o, -1^o with σ_{ij} =0.1° for all times t_i and radar tracks j. The radar noise standard deviation is consistent with levels found in modern radar systems. Since the radar results contain truth, i.e., a target moving with constant bearing of 0^o a good association algorithm will establish that there is a firm correlation between the ESM data and the 0^o bearing track. Figure 1 plots the probability the association algorithms establish a firm association between ESM data and the radar measurements. The fuzzy association algorithm results are given by the curve marked with o's and the TW results are indicated by the curve marked with + 's. The vertical axis indicates probability of firm correlation and the horizontal axis the number of data points necessary to establish that level of probability.

The fuzzy association algorithm results are always superior to the TW-algorithm. At ten data points the fuzzy algorithm has established a 65% probability of firm correlation, between the ESM data and the 0° radar track. The TW-algorithm requires about 24 points to establish the same level of probability of FCT. The fuzzy algorithm establishes an 80% probability of FCT by the 12th data point, whereas the TW-algorithm requires about 30 points to reach the same level of success. The fuzzy algorithm reaches 90% probability of FCT at 20 data points and the TW-algorithm at about the 38th point. So the fuzzy algorithm establishes high probabilities of firm correlation with between 1/3 to 1/2 the data required by the TW-algorithm. In this sense the fuzzy algorithm is 2 to 3 times faster than the TW-algorithm. Also, this is a difficult example for any association algorithm since there are two additional radar measurements within one noise standard deviation. The results are only slightly inferior to the noiseless case found in reference [1]. The ability of the fuzzy algorithm to make high quality decisions with much less data than the TW-algorithm is significant since real data is frequently sparse and intermittent.

Figure 2 presents results for the radar example $\mu = 2^o,\ 1^o,\ -1^o$ with σ_{ij} =0.1° for all times t_i and radar tracks j. The radar noise standard deviation is consistent with levels found in modern radar systems. Since the radar results do not contain truth, i.e., a target moving with constant bearing of 0^o, a good association algorithm will establish that the ESM data is firmly not correlated with the radar tracks.

Figure 2 plots the probability the association algorithms establish that the ESM and radar data are firmly not correlated. By the 8th data points the fuzzy algorithm has reached a 50% probability of FNT, whereas the TW-algorithm never exceeds that probability. For this example, the fuzzy algorithm is 6 times faster than the TW-algorithm, i.e., it reaches the TW-algorithm's maximum probability of FNT with 1/6 the data. The fuzzy algorithm has 75 and 90% probabilities of FNT by the 13th and 20th data points, respectively. The results are only slightly inferior to the noiseless case found in reference [1]. Thus once again, the fuzzy algorithm makes a high quality decision long before the TW-algorithm.

3.2 Association of Data with Random Dropout

In this subsection, ESM and radar data are associated for a single emitter and three noisy radar measurements with the same noise standard deviation as in subsection 3.1. Unlike the results in 3.1, there have been random dropouts of both ESM and radar data points. In Figures 3 and 4 that follow, radar results for Figure 1 have been used, i.e., 0, -1, 1 with 0.1° standard deviation Gaussian noise added. The vertical axis always measures the probability of FCT and the horizontal axis time (time is in arbitrary units). The horizontal axes in Figures 1 and 2 are labeled with "No. of Data Points", but the time allowed for data point accumulation is the same as in Figures 3 and 4. If the horizontal axis in Figure 1 is thought of as measured in units of time then direct comparison between Figures 1, 3 and 4 is possible. The results from the fuzzy association algorithm are marked with o's and the TW-algorithm by +'s.

In Figure 3 the probability of detection of ESM points is 90%. In this case as observed before the fuzzy algorithm reaches a 70% probability of FCT, between 2 and 3 times faster than the TW-algorithm, and 90% probability of FCT a little more than 2 times faster than the TW-algorithm. The reduction in probability of detection of the ESM data by 10% has little effect on the fuzzy association algorithm, but the TW-algorithm is showing significant deterioration.

In Figure 4 the probability of detection of ESM points is 70%. Once again the fuzzy association algorithm establishes firm correlation between the ESM and the 0° radar track more than 2 times faster than the TW-algorithm.

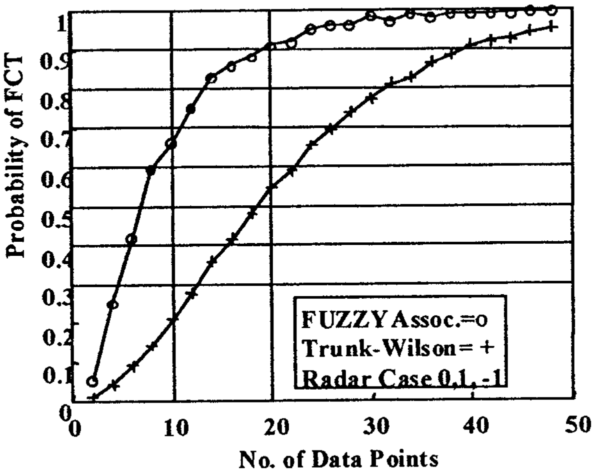

Figure 1: Noisy radar

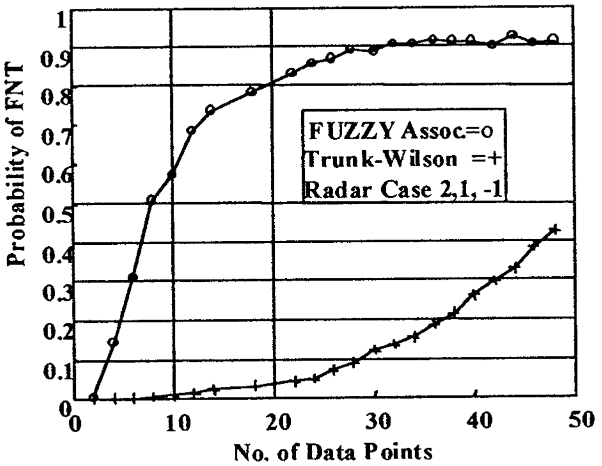

Figure 2 : Noisy radar

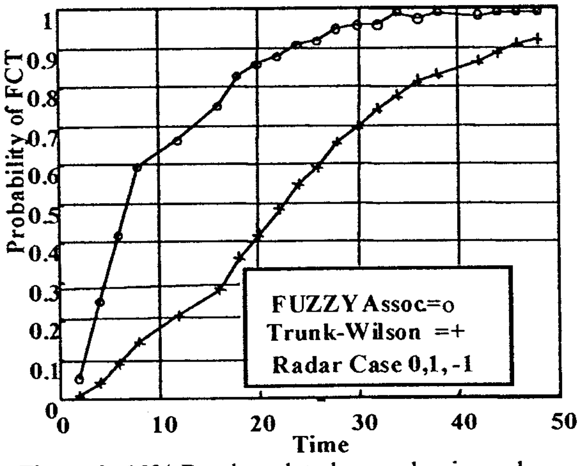

Figure 3: 10% Random data loss and noisy radar

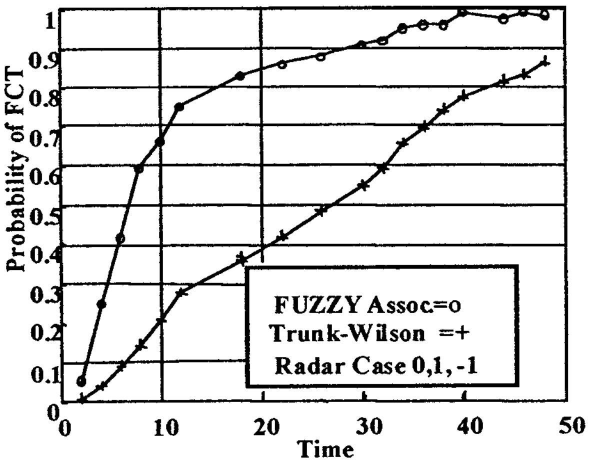

Figure 4: 30% Random data loss and noisy radar

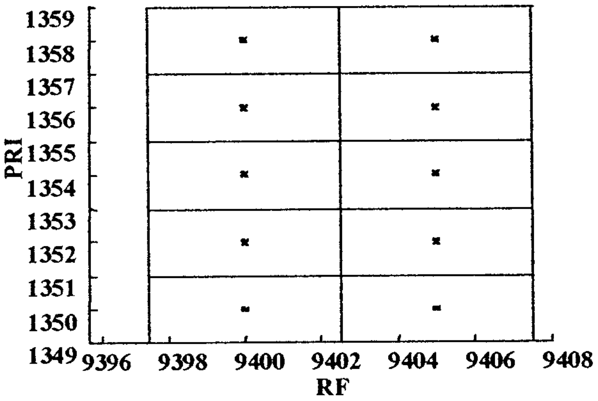

Figure 5: 10 closely spaced emitters

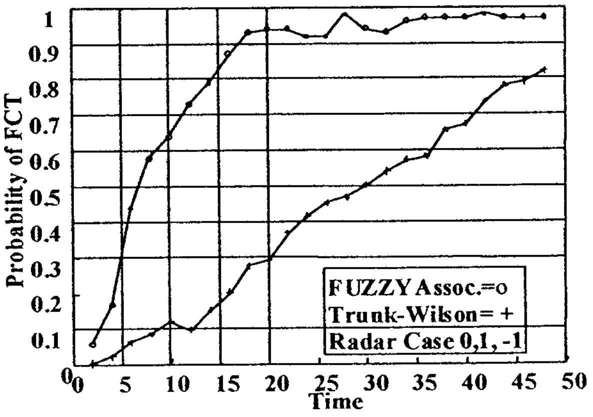

Figure 6: Noisy Radar and 100% detection

3.3 Multiple Closely Spaced Emitters

In this section the results are presented for 10 closely spaced emitters. In each case there are three distinct radar measurements, $\mu = 0^o,\ 1^o,\ -1^o$. In addition to bearing data for the multitarget case, the fuzzy clustering algorithm is used to cluster on RF and PRI, forming a RF-PRI window. The bearing for each emitter is then associated with the radar data in the same way as in the single emitter cases. In Figures 6 and 7, the ESM cluster that predominantly contains the emitter with constant bearing of 0^o has been associated with the radar data. It is important to observe that both the fuzzy association algorithm and the TW algorithm use the RF-PRI fuzzy clustering results. Since the TW algorithm is not capable of deinterleaving data it requires a preclustering operation and the fuzzy clustering algorithm has been shown to be effective for deinterleaving [4].

In Figure 5 the noiseless RF and PRI values are plotted for the 10 emitters. The rectangles are centered at the noiseless values in each case and have sides of length equal to the resolution limit of the simulated ESM system for both RF and PRI. The RF and PRI resolutions are measured relative to the appropriate axes in each case. In Figure 5 the emitter with a 0° bearing is confined to the box in the lower left corner centered at RF=9400 and PRI=1350. The algorithm was applied to a much easier 10-emitter example with much greater separation in the RF-PRI plain in reference [8].

Figure 6 presents results for the radar example $\mu = 0^o,\ 1^o,\ -1^o$ and 10 emitters as pictured in Figure 5. Since the radar results contain truth, i.e., a target moving with constant bearing of 0^o a good association algorithm will establish that there is a firm correlation between the ESM data and the 0^o bearing track. Figure 6 plots the probability the association algorithms establish a firm association between ESM data and the 0° radar measurement. The data is preclustered in RF and PRI as points come in, only then does association take place. This class of simulation is more sophisticated in the sense that the initial clustering operation can fail and points associated with the proper emitter can be neglected or points not associated with it included. The fuzzy association algorithm results are given by the curve marked with +'s and the TW results are indicated by the unmarked continuous curve. The vertical axis indicates probability of firm correlation and the horizontal axis the number of data points necessary to establish that level of probability.

The results for Figure 6 are fairly close to those of Figure 1 for the fuzzy algorithm which establishes high probabilities of FCT. This is interesting since with 10 emitters there is an opportunity for bearing points to be misclustered in two ways. One kind of error that can arise is that bearing points from the wrong emitter are assigned to the emitter of interest producing a significant outlier. The second kind of error relates to bearing points for the emitter of interest being assigned to another emitter reducing the sampling rate. The first kind of error will generally produce outliers and the second kind will make the data more intermittent. Probability augmented superclustering should be useful in dealing with both classes of errors [8]. In this simulation the fuzzy association algorithm gives performance similar to that of the single emitter case (Figure 1). The TW-algorithm experienced significant deterioration, e.g., at 20 and 48 data points it dropped from 55 to 30% and 95% to 80%, respectively. So it appears that the TW-algorithm is sensitive to errors of the first and second kind. This is to be expected since the TW-algorithm does not have the ability to deal with outliers and missed data that the fuzzy algorithm has.

In Figure 7 the same 10 emitter calculation is considered as in Figure 6 except the probability of detection of ESM points is 90%. In this case as observed before the fuzzy algorithm reaches a 70% probability of FCT, between 3 and 4 times faster than the TW-algorithm, and 90% probability of FCT at 18 data points, whereas the TW-algorithm has a probability of FCT less than 80% for the first 48 points. The reduction in probability of detection of the ESM data by 10% has little effect on the fuzzy association algorithm, but the TW-algorithm is showing significant deterioration.

In Figure 8 the probability of detection of ESM points is 70%. The fuzzy association algorithm deteriorates little under the additional loss of data; its results are similar to those found in Figures 1, 6, and 7. The TW-algorithm has experienced significant deterioration. In Figure 7 for 90% probability of detection it had nearly 80% probability of FCT at 48 data points, for a 70% detection rate its probability of FCT is less than 60%. Once again the fuzzy association algorithm maintains its performance even though the TW-algorithm experiences significant deterioration.

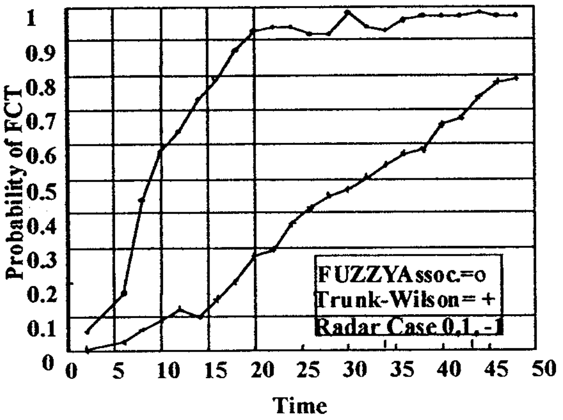

Figure 7: Noisy Radar and 90% detection

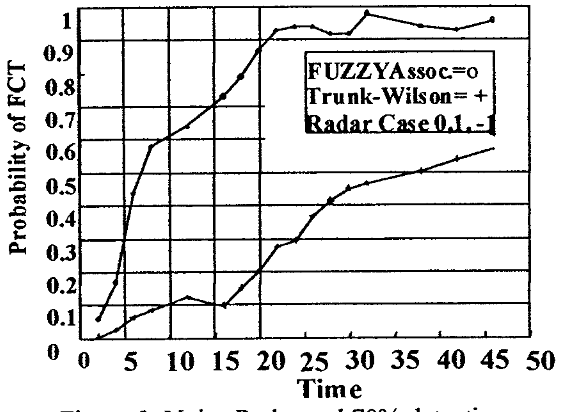

Figure 8: Noisy Radar and 70% detection

4. Sliding Windows, Performance And Computational Complexity

As discussed in subsection 2.1 and also in references [1,8] underlying the fuzzy association algorithm is fuzzy clustering. Roughly speaking the larger the number of points used in clustering, the better the algorithm's performance, because of improved cluster center estimation. There can be pathological cases where this is not true, but they are rare. Also, the algorithm's CPU time requirements increase when larger data sets are clustered assuming the algorithm is initialized in the same way. A technique for reducing CPU requirements, while maintaining performance will now be explored.

The algorithm experiences a significant speed-up if a small amount of performance is sacrificed. One method of doing this is the introduction of a sliding window. If a window of size w is used then for $n<w$ all n points are used in fuzzy clustering and superclustering operations. If $n \geq w$ then only w points are used. This results in a significant savings in CPU time and small loss in performance as shown in Figure 9.

Figure 9 is the probability of FCT versus the number of data points for the closely spaced 10-emitter example of Figure 5. A sliding window in time has been applied to the ESM bearing data. This figure presents results for four different sliding window lengths. The sliding window lengths and the associated symbols they are marked with in the figure are 40 points (*), 30 points (X), 20 points (o), and 10 points (+). The fifth curve given in terms of dot-dashes represents the results for the TW-algorithm for a 48-point window.

All four windowed fuzzy association calculations give performances superior to the TW-algorithm. For a window of length w for $n<w$ the algorithm gives performance similar to the 48 point window of Figure 6. For $n \geq w$ there is a slight deterioration in performance. The 30 and 40 point windows show the least performance loss and give results not far removed from those of Figure 6.

The introduction of a sliding window results in a significant reduction in CPU time requirements for fuzzy clustering and superclustering. The maximum expected CPU times for the 10, 20, and 30 point windows are 1/5, ½, and ¾ of that required by the 40 point window calculation. The effects of smaller windows and other approximations will be explored in the future.

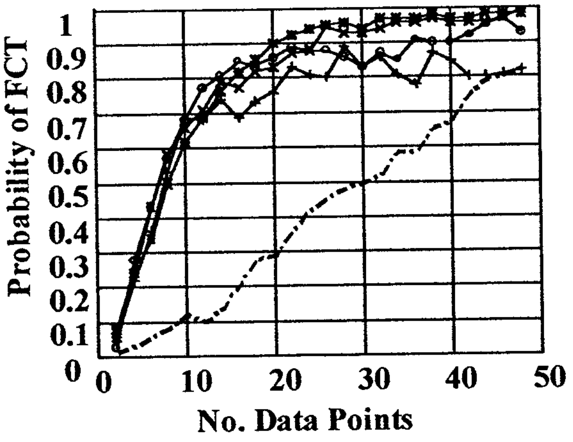

Figure 9: Sliding Windows: 40 pts.(*), 30 pts (X), 20 pts (o), 10 pts (+), and TW alg. (.-.-).

5. Conclusions

A fuzzy logic algorithm for clustering and associating data measured on different sensors has been developed. It has been compared to the Trunk-Wilson (TW) association algorithm, a Bayesian philosophy algorithm. In simulations in which noisy radar data contained truth, the fuzzy association algorithm establishes a firm correlation with 1/3 to 1/2 the data required by the TW-algorithm. When the noisy radar data did not contain truth, the fuzzy algorithm outperformed the TW- algorithm with only 1/6 the data. When the ESM and radar systems randomly fail to measure up to 30% of the data points the fuzzy association algorithm shows little deterioration and is always better than the TW-algorithm. The TW-algorithm showed marked deterioration when data points are randomly lost or there are multiple closely spaced emitters. Introduction of sliding windows significantly reduces CPU requirements while offering little deterioration in performance. The fuzzy association algorithm's ability to make correct decision with much less data than the TW-algorithm is crucial since ESM data is generally sparse, intermittent, and noisy. Finally, the fuzzy association algorithm should be applicable to many different multisensor problems requiring high quality decisions.

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