Visualization in optimization with MATHEMATICA

Predrag S. Stanimirović,
Marko D. Petković,
Milan Lj. Zlatanović

Faculty of Sciences and Mathematics
University of Niš, Serbia

peckois@ptt.yu
dexterofnis@gmail.com
zlatmilan@yahoo.com

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Outline of the talk

1. Program package MATHEMATICA
   - Introduction
   - Main capabilities of MATHEMATICA
2. Visualization of 2D and 3D linear programming problems
   - Linear programming problem
   - 2D case
   - 3D case
   - Properties of the feasible set
3. Conclusion, future work, references
   - Conclusion
   - Future work
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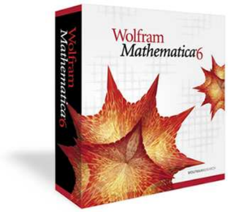
Introduction



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- It is mainly known as a computer algebra system (CAS), but it has various other features for technical computing.
- Made by Stephen Wolfram. First realized in 1988 (version 1.0). Last version (6.0) realized in 2007.
- MATHEMATICA is used today throughout the sciences–physical, biological, social, and other–and counts many of the world's foremost scientists among its enthusiastic supporters.
- MATHEMATICA is also heavily used in education, and there are now many hundreds of courses–from high school to graduate school–based

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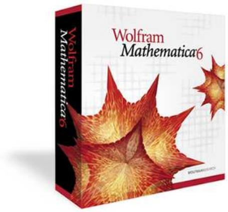

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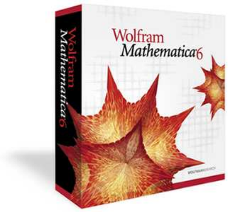

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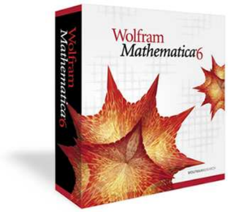

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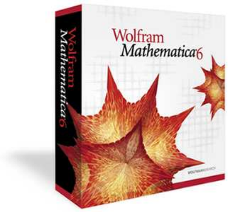

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Main capabilities of MATHEMATICA

- All the computation you need (huge collection of algorithms in a single system).
- Symbolic computation (work with the exact numbers, expressions, terms, symbolic integration, simplification, etc.).
- Any precision numerical computation (MATHEMATICA's numerics support any precision or number size across all functions).
- Work with any data (Provides the support for the various data formats including the image, sound, etc.)
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- Visualize anything (Functions or data, discrete objects, diagrams, images, annotations, etc).

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Outline of the talk

Linear programming problem

Linear programming (LP) problem in general form is given by:

$$\max(\min)\ f(x) = c_1x_1 + \cdots + c_nx_n$$

$$N_i^{(1)}: \quad \sum_{j=1}^{n} a_{ij}x_j \le b_i, \quad i = 1, \ldots, p$$

$$N_i^{(2)}: \quad \sum_{j=1}^{n} a_{ij}x_j \ge b_i, \quad i = p+1, \ldots, q$$

$$J_i: \quad \sum_{j=1}^{n} a_{ij}x_j = b_i, \quad i = q+1, \ldots, m$$

$$x_j \ge 0, \quad j \in \mathcal{J} = \{1, \ldots, s\}, \quad s \le n$$

- Solutions $x = (x_1, \ldots, x_n)$ of the constraints system are called feasible solutions. Set of all feasible solution Ω_P is called feasible set.
- In the geometrical interpretation, feasible set Ω_P is one polytope (simplicial complex) in $\mathbb{R}^n$.

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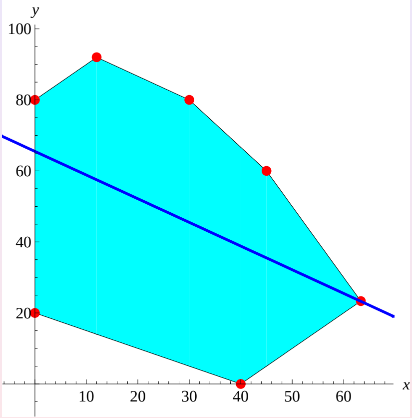
2D case

Consider the following 2D linear programming problem:

$$\begin{aligned} \max \quad & f(x,y) = 8x + 12y \\ \text{s.t.} \quad & 8x + 4y \leq 600 \\ & 2x + 3y \leq 300 \\ & 4x + 3y \leq 360 \\ & 5x + 10y \geq 600 \\ & x - y \geq -80 \\ & x - y \leq 40 \\ & x, y \geq 0. \end{aligned}$$



3D case

Consider the following 3D linear programming problem:

$$
\begin{aligned}
\max \quad & f(x,y,z) = x + y + z \\
\text{s.t.} \quad & x + y + z \le 1 \\
& 2x + y \le 1 \\
& x + y \le 1 \\
& 2y + z \le 4/3 \\
& z + x \le 2/3 \\
& y \le 3/5 \\
& z \le 1/2 \\
& x, y, z \ge 0.
\end{aligned}
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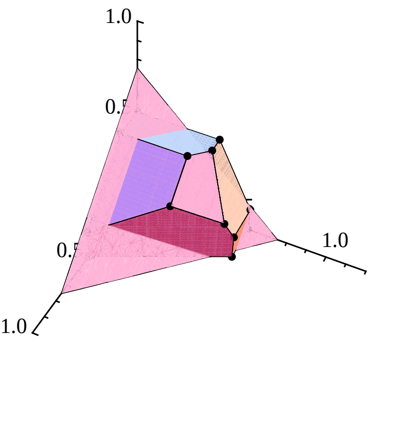

Properties of the feasible set

It is intuitively clear, from the visualizations, that following properties are true.

Theorem

Feasible set Ω_P convex polytope in $\mathbb{R}^n$.

Theorem

Let Ω_P is bounded. There exists $\inf_{x \in \Omega_P} f(x)$, and it is reached in some extremal point $x^* \in \Omega_P$. Set of the optimal solutions $\Omega_P^* = \{x | x \in \Omega_P, f(x) = f(x^*)\}$ is convex.

Theorem

Feasible solution $x \in \Omega_P$ is the basic solution if and only if it is extremal point of the set Ω_P.

Conclusion: Optimal solution is one of the most $\binom{n}{m}$ extremal points (basic solutions) of the set Ω_P. This fact is fundamental for the geometrical and simplex method.

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Outline of the talk

Conclusion

- We develop the symbolic implementation and visualization of geometrical method for solving 2D and 3D linear programming problems.
- Software is made in MATHEMATICA 5.2 and MATHEMATICA 6.0, it is free and can be downloaded from the web site:

  http://tesla.pmf.ni.ac.yu/people/dexter/freewaresoftware.htm

This software can be used in:

- Educational purpose (teaching LP university and talented high school students).
- Scientific purpose (construction of new methods and heuristics for solving LP)

Example: Minimal angles method (Stojković, Stanimirović 2002).

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Future work

Some directions for the future considerations:

- Visualization of the simplex method.
- Visualization of the primal-dual interior point methods.
- Visualization of the multicriteria optimization methods including the set of Pareto optimal solutions.
- Visualization of the unconstrained nonlinear programming methods (Simplex I, Simplex II, parabola method, gradient methods, etc.)
- Visual consideration of the game theory.

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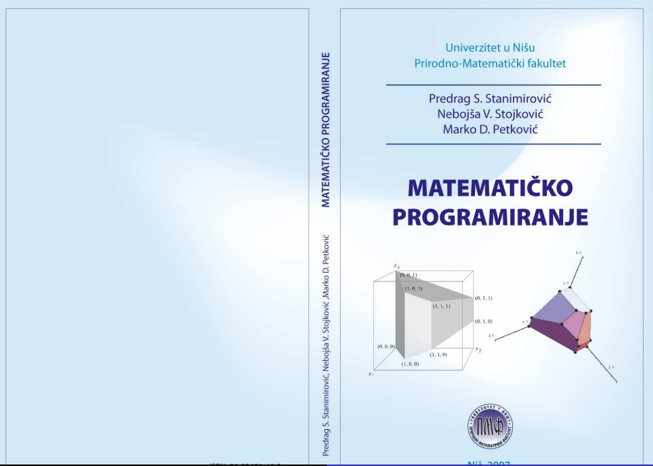

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Thanks for attention!