International Journal of Modern Physics C, Vol. 12, No. 10 (2001) 1525–1535

ON TELEGRAPH COUPLED MAP LATTICE AND ITS APPLICATIONS

E. AHMED

Mathematics Department, Faculty of Sciences, Al-Ain P. O. Box 17551 U. A. E.
Mathematics Department, Faculty of Sciences, Mansura, 35516, Egypt

H. A. ABDUSALAM* and E. S. FAHMY

Mathematics Department, Faculty of Sciences, Cairo University, Giza, Egypt
**hosny@math-sci.cairo.eun.eg*

Received 2 September 2001
Revised 10 September 2001

In the present paper, we will study the general telegraph coupled map lattice and finite amplitude instability will be discussed. The persistence in telegraph reaction diffusion equation (TRD) will be studied in two dimensional systems and we show the critical size effect where phenomena persist only if the domain is large enough. Some applications are introduced and we made a simulation on the small world network, which is more realistic than the ordinary lattices in many cases.

Keywords: Telegraph reaction diffusion equation; coupled map lattice; persistence; finite amplitude instability.

1. Introduction

The standard coupled map lattice (CML)[1] depends on the familiar reaction diffusion equation

$$\frac{\partial u(x,t)}{\partial t} = D\frac{\partial^2 u(x,t)}{\partial x^2} + f(u)\,. \tag{1}$$

A basic weakness of this equation is that the flux j reacts simultaneously to the gradient of u consequently an unbounded propagation speed is allowed. This manifests itself in many solutions to Eq. (1), e.g. (if $f = 0$),

$$u(x,t) = \frac{1}{\sqrt{4\pi Dt}}\, e^{-x^2/4Dt}\,, \quad u(x,0) = \delta(x) \quad \text{i.e} \quad u(x,t) > 0\,\forall\, x\,, \quad \forall\, t > 0\,.$$

This is unrealistic specially in biological and economical systems where it is known that propagation speeds are typically small. To rectify this weakness Fick's law is replaced by:

$$j + \tau\frac{\partial j}{\partial t} = D\frac{\partial u}{\partial x}$$

and the resulting telegraph diffusion equation is:

$$\tau\frac{\partial^2 u}{\partial t^2}+\frac{\partial u}{\partial t}=D\frac{\partial^2 u}{\partial x^2}\,.$$

The corresponding telegraph reaction diffusion (TRD)[2] is:

$$\tau\frac{\partial^2 u}{\partial t^2}+\left(1-\tau\frac{df}{du}\right)\frac{\partial u}{\partial t}=D\frac{\partial^2 u(x,t)}{\partial x^2}+f(u)\,. \tag{2}$$

The time constant τ can be related to the memory effect of the flux j as a function of the distribution u as follows[3]: Assume

$$j(x,t)=-\int_0^t K(t-t')\frac{\partial u(x,t')}{\partial x}\,dt' \tag{3}$$

hence

$$j+\tau\frac{\partial j}{\partial t}=-\tau K(0)u(x,t)-\int_0^t\left(\tau\frac{\partial K(t-t')}{\partial t}+K(t-t')\right)\frac{\partial u}{\partial x}\,dt'\,.$$

This equation is equivalent to the telegraph equation if $K(t)=(D/\tau)e^{-t/\tau}$. This lends further support that TRD is more suitable for economic and biological systems than the ordinary diffusion equation since it is known that we take our decisions according to our previous experiences so memory effects are quite relevant. Further evidence comes from the work of Chopard and Droz,[4] where they have shown that, starting from discrete time and space, the continuum limit does not give the standard reaction diffusion but the telegraph one has. Therefore the study of telegraph coupled map lattice (TCML), i.e., the coupled map lattices corresponding to telegraph reaction diffusion equation is important: The general TCML is:

$$\left.\begin{aligned}
v_j^{t+1}&=u_j^t\,,\\
u_j^{t+1}&=2u_j^t-\nu_j^t-\alpha\left(\frac{1}{\tau}-f'(u_j^t)\right)(u_j^t-\nu_j^t)\\
&\quad+\left(\alpha\frac{D}{\tau}\right)[u_{j-1}^t+u_{j+1}^t-2u_j^t]+\frac{\alpha f(u_j^t)}{\tau}
\end{aligned}\right\} \tag{4}$$

where α is a constant small parameter to avoid overflow problems.

In previous work,[5] the telegraph reaction diffusion is studied in one and two spatial dimensions. Some exact and approximate results are obtained. A coupled map lattice corresponding to the spatial prisoner's dilemma game is constructed and studied in the weak diffusion limit. A formula is derived for Lyapunov exponents and it is shown that periodic solutions are dense in the weak coupling regime and that this system is structurally stable.

In the present work, we study the asymptotic stability, finite amplitude instability of telegraph coupled map lattice and the persistence of the TRD equation. Also, some application are introduced and the TRD for interacting systems have been studied.

2. Asymptotic Stability and Finite Amplitude Instability of Telegraph Coupled Map Lattices

It is straightforward to study the asymptotic stability of TCML, Eq. (4) and the result is that the homogeneous steady state $u_j^t = u$ where $f(u) = 0$ is stable if

$$\left.\begin{array}{c} |c| < 2\,, \quad |b| + 1 > |c|\,, \quad b = -1 + \alpha\left(\frac{1}{\tau} - f'\right) \\ c = 2 - \alpha\left(\frac{1}{\tau} - f'\right) + \alpha u f'' + \alpha\frac{f'}{\tau} \end{array}\right\}. \tag{5}$$

Now, finite amplitude instability (FAI) will be discussed. For simplicity, it will be studied for partial differential equations corresponding to the CML. It is known that the singularities of the PDE are contained in those of the corresponding discretized system.[6] Here we will assume $f''(0) \neq 0$. Thus we consider

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u(x,t)}{\partial x^2} + f(u)\,, \quad f(0) = 0\,, \quad u(0,t) = u(1,t) = 0\,. \tag{6}$$

$u = 0$ is a steady state solution so expanding near it we set

$$u = \sum_{m=1} \varepsilon^m v_m(t', t'', x)\,, \quad \frac{\partial}{\partial t} = \frac{\partial}{\partial t'} + \varepsilon\frac{\partial}{\partial t''}\,, \quad f'(0) = \lambda_0 + \varepsilon\lambda_1\,.$$

Substituting in Eq. (6) and equating terms $O(\varepsilon)$ one gets

$$v_1(t', t'', x) = \sum_{l=1} a_l(t'') \sin(\pi l x) \exp[(1 - l^2)\pi^2 t']\,.$$

Let $\lambda_0 = \pi$ and consider the equation $O(\varepsilon^2)$, we set the secular term (independent of t') equal to zero

$$\frac{da_1}{dt''} = \lambda_1 a_1 + \frac{b' f''(0)}{2} a_1^2\,, \quad b' = \frac{\int_0^1 \sin^3 \pi x\, dx}{\int_0^1 \sin^2 \pi x\, dx}\,. \tag{7}$$

Thus the conditions for FAI are:

$$f''(0) > 0\,, \quad \lambda_1 < 0 \quad \text{and} \quad a_1(0) > \frac{-2\lambda_1}{b' f''(0)}\,. \tag{8}$$

Generalizing to the telegraph equation (2) we obtained that the conditions for FAI are identical to those for the ordinary diffusion equation. Hence applying for the case of 2×2 game where $f(p) = p(1-p)(\alpha + \gamma p)$ then FAI is possible if $\gamma > \alpha$.

For systems of two partial differential equations,

$$\left.\begin{array}{c} \frac{\partial u_1}{\partial t} = D_1\frac{\partial^2 u_1(x,t)}{\partial x^2} + f(u_1, u_2)\,, \quad \frac{\partial u_2}{\partial t} = D_2\frac{\partial^2 u_2(x,t)}{\partial x^2} + g(u_1, u_2) \\ f(0,0) = g(0,0) = 0\,, \quad u_1(0,t) = u_1(1,t) = u_2(0,t) = u_2(1,t) = 0 \end{array}\right\}. \tag{9}$$

Expanding near the steady state solutions $u_1 = u_2 = 0$ we got

$$u_1 = \sum_{m=1} \varepsilon^m v_m(t', t'', x)\,,$$

$$u_2 = \sum_{m=1} \varepsilon^m \omega_m(t', t'', x)\,, \quad \frac{\partial}{\partial t} = \frac{\partial}{\partial t'} + \varepsilon \frac{\partial}{\partial t''}\,,$$

$$f_1 := \frac{\partial f}{\partial u_1}(0,0) = \lambda_{11} + \varepsilon\lambda_{12}\,, \quad g_1 = \mu_{11} + \varepsilon\mu_{12}\,, \quad g_2 = \mu_{21} + \varepsilon\mu_{22}\,.$$

After some tedious calculations we got the following conditions for FAI in Eq. (9):

(i) $(\pi^2 D_1 - \lambda_{11})(\pi^2 D_2 - \mu_{21}) - \lambda_{21}\mu_{11} = 0\,.$

(ii) $\lambda_{12} + \varkappa\lambda_{22} = \dfrac{\mu_{12}}{\varkappa} + \mu_{22} < 0$, where $\varkappa = \dfrac{\pi^2 D_1 - \lambda_{11}}{\lambda_{21}}\,.$

(iii) $\dfrac{f_{11}}{2} + \varkappa f_{12} + \left(\dfrac{\varkappa^2}{2}\right) f_{22} = \dfrac{g_{11}}{2\varkappa} + g_{12} + \left(\dfrac{\varkappa}{2}\right) g_{22} > 0\,.$

(iv) $a_1(0) > -\dfrac{\lambda_{12} + \varkappa\lambda_{22}}{b'(f_{11}/2 + \varkappa f_{12} + (\varkappa^2/2) f_{22})}\,.$

where b' is defined in Eq. (7).

For predator prey system

$$f = ru_1(1 - u_1) - u_1 u_2\,, \quad g = -au_2 + u_1 u_2\,,$$

condition (ii) is not satisfied. For Belousov–Zhabotinsky system

$$g = u_1 - u_2$$

hence condition (iii) is not satisfied. Similarly for FitzHugh–Nagumo model where

$$g = c(u_1 - bu_2 + a)\,,$$

a, b and c are constants. A model satisfying the above conditions is given by:

$$f = u_1 + u_2 + b_1 u_1 u_2\,, \quad g = u_1 + u_2 + b_1 u_1 u_2\,, \quad b_1 > 0\,, \quad b_2 > 0\,, \tag{10}$$

such that

$$(\pi^2 D_1 - 1)(\pi^2 D_2 - 1) = 1\,, \quad \varkappa = (\pi^2 D_1 - 1) > 0\,, \quad \mu_{12} = \varkappa\lambda_{12} < 0\,,$$

$$\mu_{12} = \varkappa\lambda_{12} < 0\,, \quad (\pi^2 D_1 - 1) b_1 = b_2 \quad \text{and} \quad a_1(0) > -\frac{\lambda_{12} + \varkappa\lambda_{22}}{\varkappa b_1}\,.$$

Now the case $f'(0) = 0$ is considered.[6] Linearizing around the solution $u(x,t) = 0$, i.e., let $u(x,t) = v(x,t)$, linearize in ν then $\partial v/\partial t = \partial^2 v/\partial x^2 + \lambda\nu$, $\lambda = f'(0)$. Set $u = (\sum_l a_l \sin(\pi l x))\exp(\sigma t)$, then bifurcation points are given at $\lambda = (\pi l)^2$, $l = 1, 2, \ldots$.

Studying the stability of the first bifurcation point $\lambda = \pi^2$ using Matkowsky two time nonlinear stability analysis, one defines $\lambda = \pi^2 + \lambda_0 \varepsilon^2$, ε is a small parameter. Decompose the time into fast t' and slow τ then

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t'} + \varepsilon^2 \frac{\partial}{\partial \tau},$$

expand u in powers of ε^2

$$u \simeq \varepsilon v_1 + \varepsilon^3 v_3 + \cdots,$$

(notice that $f'(0) = 0$) then substitute in Eq. (6) one gets

$$v_1(x, t', \tau) = \sum_l a_l(\tau) \sin(\pi l x) \exp(1 - l^2)\pi^2 t'.$$

Substituting into the cubic term and setting the constant term in t' equal to zero one finally gets

$$\frac{da_1}{d\tau} = \lambda_0 a_1 + b(a_1)^3, \quad b = \frac{\pi^2 f'''(0) \int_0^1 \sin^4 \pi x\, dx}{6 \int_0^1 \sin^2 \pi x\, dx}.$$

Thus a nonlinear (finite amplitude) instability arises if

$$\lambda_0 < 0 \quad \text{and} \quad |a_1(0)| > \frac{\sqrt{|\lambda_0|}}{b}. \tag{11}$$

3. On Persistence in Telegraph Reaction Diffusion Equation

Definition 1. A dynamical system is persistent if $\forall \underline{x}(0) > 0$ then

$$\lim_{t \to \infty} \inf x_i(t) > 0 \ \forall i = 1, 2, \ldots, n.$$

To study persistence in TRD the following result is crucial (Ref. 7 with slight modifications):

Theorem 1. Suppose that $f(\mathbf{r}, u)$ is Lipschitz in $\mathbf{r}$ and continuously differentiable in u with

$$\frac{\partial f}{\partial u} \le 0 \quad \text{for } u \ge 0, \quad f(\mathbf{r}, u) \le 0 \quad \text{if } u \ge l \tag{12}$$

for some constant l, and $f(\mathbf{r}, 0) > 0$ at some point in the domain Ω then the problem

$$\frac{\partial u}{\partial t} = D \frac{\partial^2 u(x,t)}{\partial x^2} + u f(\mathbf{r}, u) \quad \text{in } \Omega \in (0, \infty) \tag{13}$$

with Dirichlet or Neumann boundary conditions has a unique positive steady state u^{ss}, which is a global attractor for nontrivial non-negative solutions (hence the system (13) is persistent) if the following problem has a positive eigenvalue σ

$$\sigma u = D \frac{\partial^2 u(x,t)}{\partial x^2} + u f(\mathbf{r}, 0) \quad \text{in } \Omega$$

with the same boundary conditions as Eq. (13).

Proposition 1. *The conditions of Theorem* 1 *implies the persistence of the TRD equation* (2) *with f replaced by uf.*

Proof. Linearizing around $u = 0$ and let $u = \exp(\sigma t)u(\mathbf{r}, 0)$ one gets

$$\sigma = \frac{-1 + \tau f(0) \pm \sqrt{(1 - \tau f(0))^2 + 4\tau\gamma}}{2\tau},$$

$$\gamma = f(0) - D\left(\frac{\pi^2}{L_1^2} + \frac{\pi^2}{L_2^2}\right).$$

Thus at least one of the two values of σ is positive if $\gamma > 0$. □

Now Theorem 1 and Proposition 1 is applied to some two-dimensional systems. The domain Ω is chosen to be the rectangle $x \in [0, L_1]$, $y \in [0, L_2]$. The first is spatial games where f in Eq. (13) is given by:

$$f = (1 - u)(\alpha + \gamma u), \quad u \in [0, 1]. \tag{14}$$

Then the system (13) with f given by Eq. (14) is persistent if

$$\alpha \geq |\gamma| \quad \text{and} \quad \alpha > D\left(\frac{\pi^2}{L_1^2} + \frac{\pi^2}{L_2^2}\right).$$

This result shows the critical size effect where a phenomena persist only if the domain is large enough.

The function f for the mean field contact process is[8]:

$$f = \beta(1 - u) \tag{15}$$

and the system is persistent if $\beta > D(\pi^2/L_1^2 + \pi^2/L_2^2)$.

The function f for the spread of spruce budworm is:

$$f = r\left(1 - \frac{u}{K}\right) - \frac{u}{1 + u^2}, \quad u \in [0, 1] \tag{16}$$

and the system is persistent if $r > D(\pi^2/L_1^2 + \pi^2/L_2^2)$.

4. Applications

As an application we have simulated TCML with logistic map $f(u) = ru(1 - u)$ on small world network (SWN) which, in many cases, is more realistic than ordinary lattices. We begin by reviewing the basics of SWN: small world networks[9,10] is a model proposed for social networks. It is a one-dimensional ring plus shortcuts joining some random sites. Here we consider shortcuts with length $k = 1$. Let ϕ be the average number of shortcuts/bond in the lattice hence, for large number (L) of sites in the lattice, the probability that two random sites are connected by a shortcut is $\psi \simeq 2k\phi/L$. Naturally the critical concentration of this graph p_c is smaller than the one for the ring $p_c = 1$. To derive the new p_c the work of Moore and Newman will be used. Build the lattice starting from a connected local cluster then follow shortcuts. Let v_i $(i = 1, 2, \ldots, n)$ be the probability that a local cluster of length i is included, hence in the next step in the cluster building one has

$$\tilde{v}_i = \sum M_{ij} v_j \,, \quad M_{ij} \simeq ij\Psi N \,. \tag{17}$$

For full derivation, see Ref. 11. If the maximum eigenvalue of M_{ij} is less than one then the propagation process will eventually stop, otherwise it will propagate throughout the cluster. Thus the critical concentration corresponds to the eigenvalue one, i.e.,

$$\sum M_{ij} v_j = \lambda v_i \Longrightarrow \lambda = \Psi \sum j^2 N_j \Longrightarrow \lambda = 2\phi p \, \frac{1+p}{1-p} \,.$$

So the critical concentration is:

$$p_c = \frac{\sqrt{4\phi^2 + 12\phi + 1} - 2\phi - 1}{4\phi} \,. \tag{18}$$

For small ϕ,

$$p_c \simeq 1 - 4\phi \,.$$

Now consider the propagation across a small world network. We consider the spread of an epidemic in a population with random susceptibility; thus our work generalizes that of Moore and Newman.[11] In our simulations, $\phi = 0.05$ and shortcuts are assigned randomly from the beginning.

When the TCML with logistic $f(u)$ is simulated on SWN with $\tau = \alpha = 0.01$, we obtained (for $d = 0$) that the TCML converged to a fixed point for $r < 2.71$. For $2.71 < r < 2.79$, the system showed cycles. For $2.79 < r < 3.1$, the system is chaotic and for $r > 3.1$ there is overflow. When $d = 0.03$ we obtained fixed points for $0 < r < 1.81$, cycles for $1.81 < r < 2.71$ and chaos for $2.71 < r < 2.77$.

We then studied the CML with the forced logistic map

$$\left.\begin{aligned} f(u_t) &= u_t(1 - u_t)(r + \varepsilon \cos(\theta_t)) \\ \theta_{t+1} &= \theta_t + \frac{2\pi}{3} \end{aligned}\right\} . \tag{19}$$

For simplicity we assumed that τ is so small that TCML can be approximated by the ordinary CML. The addition of periodic forcing is important from the ecological point of view.[12] The results are shown in Fig. 1 where $r = 3.3$ and the number of lattices is $n = 50$. Again we simulated the system on SWN and the results are shown in Fig. 1, where the average Lyapunov exponent as a function of the forcing ε, the average Lyapunov is calculated by finding the Lyapunov exponent at each site then averaging over all sites. Curve A is the Lyapunov exponent for $d = 0$, curve B is the Lyapunov exponent for one-dimensional lattices (i.e., SWN with $\varphi = 0$) with $d = 0.1$. Curve C is the Lyapunov exponent for SWN ($\varphi = 0.2$) with $d = 0.1$.

Finally we studied the CML for the forced circle map.[13] The dynamics is defined by the equations

$$\begin{aligned} \theta_{t+1} &= \theta_t + \varpi + \frac{k}{2\pi}\sin(2\pi\theta_t) + \frac{c}{2\pi}\cos(2\pi\varphi_t) \mod 1 \\ \varphi_{t+1} &= \varphi_t + \varpi_1 \mod 1 \end{aligned}$$

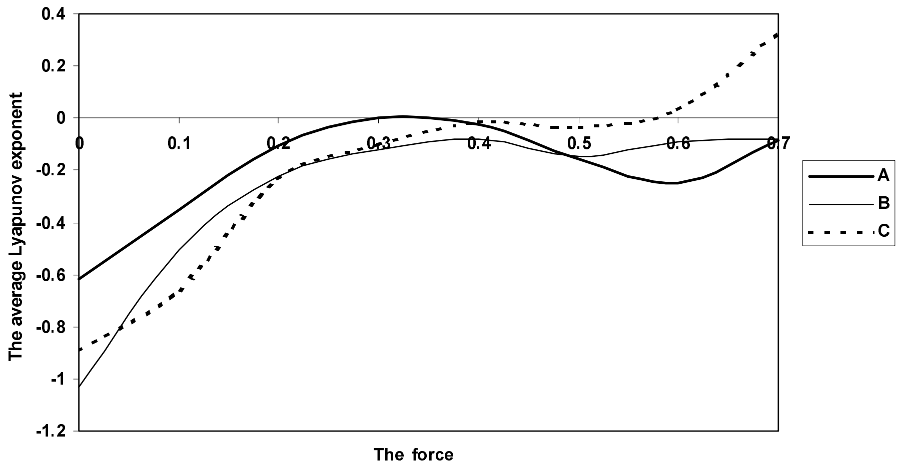

Fig. 1. The average Lyapunov exponent as a function of the forcing ε.

where θ_t and φ_t module 1 give the coordinates on the torus. The parameter ϖ is the phase shift, k denotes the strength of nonlinearity ($k > 0$), ε is the forcing amplitude, and the forcing frequency ϖ_1 is irrational. Forming the corresponding coupled map lattice on the SWN for different c our results are given in Figs. (2)–(4), where in all our simulations we have $\varpi_1 = 1/\sqrt{2}$, $n = 10$ and SWN connectivity equal 0.2.

Figure 2 shows the average Lyapunov exponent as a function of the forcing ϖ, where $k = 0.2$, $c = 0$, and $d = 0$ for curve A; $k = 0.2$, $c = 0$, $d = 0.1$ for one-

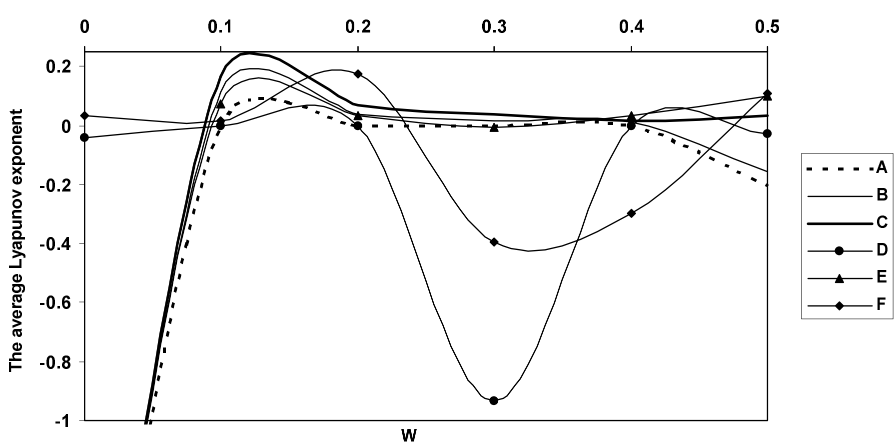

Fig. 2. The average Lyapunov exponent as a function of the forcing ϖ, where $k = 0.2$, $c = 0$, and $d = 0$ for curve A; $k = 0.2$, $c = 0$, $d = 0.1$ for one-dimensional lattice ($\varphi = 0$) for curve B; $k = 0.2$, $c = 0$, $d = 0.1$ for SWN ($\varphi = 0.2$) for curve C; $k = 0.2$, $c = 0.6$, $d = 0$ for curve D; $k = 0.2$, $c = 0.6$, $d = 0.1$ for one-dimensional lattice ($\varphi = 0$) for curve E; and $k = 0.2$, $c = 0.6$, $d = 0.1$ for SWN ($\varphi = 0.2$) for curve F.

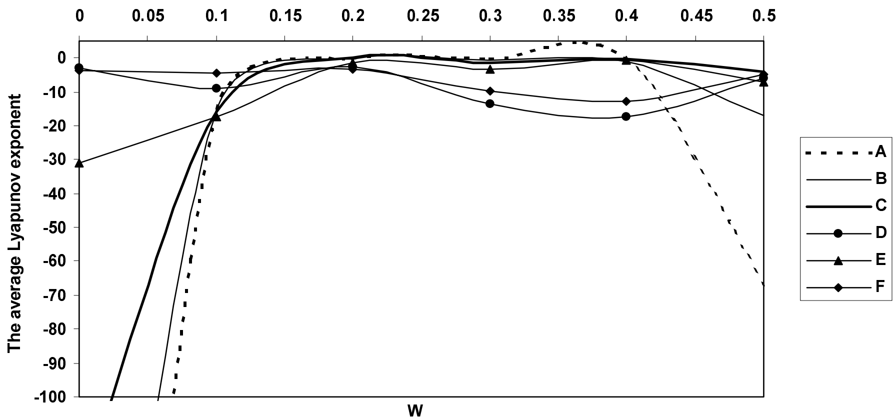

Fig. 3. The average Lyapunov exponent as a function of the forcing ϖ, for curves $A, \ldots, F$ but with $k = 1$, with the same values of c, d, φ, respectively as given in Fig. 2.

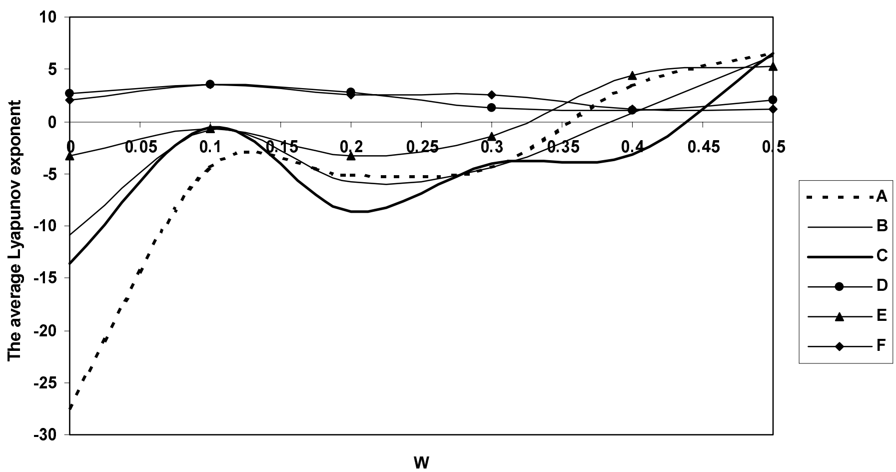

Fig. 4. The average Lyapunov exponent as a function of the forcing ϖ, for curves $A, \ldots, F$ but with $k = 2.5$ with the same values of c, d, φ, respectively as given in Fig. 2.

dimensional lattice ($\varphi = 0$) for curve B, $k = 0.2$, $c = 0$, $d = 0.1$ for SWN ($\varphi = 0.2$) for curve C; $k = 0.2$, $c = 0.6$, $d = 0$ for curve D; $k = 0.2$, $c = 0.6$, $d = 0.1$ for one-dimensional lattice ($\varphi = 0$) for the curve E; and $k = 0.2$, $c = 0.6$, $d = 0.1$ for SWN ($\varphi = 0.2$) for curve F.

Figures 3 and 4 show the average Lyapunov exponent as a function of the forcing ϖ, for curves $A, \ldots, F$ but with $k = 1$, 2.5, respectively.

5. Telegraph Reaction Diffusion Equation for Interacting Systems

For simplicity we will consider the partial differential equations. The generalization to CML is straightforward.

The TRD equations for interacting systems are:

$$\tau \frac{\partial^2 u_i}{\partial t^2} + \frac{\partial u_i}{\partial t} - \tau \sum_j \frac{\partial u_j}{\partial t} \frac{\partial f_i}{\partial u_j} = D \frac{\partial^2 u_i(x,t)}{\partial x^2} + f_i(u_1, u_2, \ldots, u_n)\,, \tag{20}$$

where $i = 1, 2, \ldots, P$. Consider the predator prey (or epidemic) system where $P = 2$,

$$f_1 = u_1(a_1 - b_1 u_2)\,, \quad f_2 = u_2(-a_2 + b_2 u_1)\,. \tag{21}$$

The system has a homogeneous steady state $u_1 = a_2/b_2$, $u_1 = a_2/b_2$. It is asymptotically stable if

$$\alpha_1\alpha_2 - \alpha_3 > 0\,, \quad \alpha_1(\alpha_2\alpha_3 - \alpha_1\alpha_4) - \alpha_3^2 > 0\,, \tag{22}$$

where

$$\alpha_1 = \frac{2}{\tau}\,, \quad \alpha_2 = \left(\frac{1}{\tau^2}\right)\left[\tau(D_1 + D_2)\left(\frac{n\pi}{L}\right)^2 + 1 - b_1 b_2 \tau^2\right],$$

$$\alpha_3 = \left(\frac{1}{\tau^2}\right)\left[(D_1 + D_2)\left(\frac{n\pi}{L}\right)^2 + \left(\frac{a_1 a_2}{b_1 b_2}\right)(-b_1 + b_2)\tau\right],$$

$$\alpha_4 = \frac{D_1 D_2 n^4 \pi^4}{L^4} + \frac{a_1 a_2}{b_1 b_2}\,,$$

where $0 \le x \le L$.

Similarly Turing instability can be studied where the system is given by:

$$f_1 = a_{11}u_1 + a_{12}u_2\,, \quad f_2 = a_{21}u_1 + a_{22}u_2\,. \tag{23}$$

And it will occur if any of the following conditions is "Not" satisfied $\forall\, k$:

(i) $\alpha_1 > 0$, (ii) $\alpha_4 > 0$, (iii) $\alpha_1\alpha_2 > \alpha_3$, (iv) $\alpha_1(\alpha_2\alpha_3 - \alpha_1\alpha_4) > \alpha_4^2$,

where

$$\alpha_1 = \frac{2 - \tau a_{11} - \tau a_{22}}{\tau}\,,$$

$$\alpha_2 = \left(\frac{1}{\tau^2}\right)[\tau(D_2k^2 - a_{22}) + \tau(D_1k^2 - a_{11}) + (1 - \tau a_{11})(1 - \tau a_{22}) - \tau^2 a_{12}a_{21}]\,,$$

$$\alpha_3 = \left(\frac{1}{\tau^2}\right)[(D_2k^2 - a_{22})(1 - \tau a_{11}) + (D_1k^2 - a_{11})(1 - \tau a_{22}) - 2\tau a_{12}a_{21}]\,,$$

$$\alpha_4 = \left(\frac{1}{\tau^2}\right)[(D_2k^2 - a_{22})(D_1k^2 - a_{11}) - a_{12}a_{21}]\,.$$

In the standard diffusion equation, minimizing α_4 with respect to k^2 gives the standard Turing conditions.[13]

6. Conclusion

We have studied the asymptotic stability and discussed the finite amplitude instability for the TCML. The work is generalized to the TRD equation and we find that, the conditions for FAI are identical to those for ordinary diffusion equation. Also, the conditions for FAI are derived in the case for systems of two partial differential equations. The persistence in TRD has been studied.

We have simulated the TCML with logistic map on SWN and also the CML for the forced circle map has been studied. The TRD equation for interacting systems is introduced and as an example we study the predator prey system and we find that, under some conditions the system has a homogeneous steady states which is asymptotically.

Acknowledgments

We are grateful to Prof. R. Cantrell and C. Cosner for sending copies of their work to us and for discussions.

References

1. K. Kaneko, *Theory and Applications of Coupled Map Lattice* (John Wiley & Sons, Ltd., 1993).
2. E. Ahmed and S. Z. Hassan, *Z. Naturforsch* **55a**, 669 (2000).
3. A. Compte and R. Metzler, *J. Phys. A* **30**, 7277 (1997).
4. B. Chopard and M. Droz, *J. Stat. Phys.* **64**, 859 (1991).
5. E. Ahmed, H. A. Abdusalam, and E. S. Fahmy, *Int. J. Mod. Phys. C* **12**, 717 (2001).
6. A. Stuart, *Siam Rev.* **31**, 192 (1989).
7. R. S. Cantrell, C. Cosner, and V. Hutson, *Proc. Soc. Edin* **123A**, 533 (1993).
8. R. Durrett, *Essentials of Stochastic Processes* (Springer, 1999).
9. D. J. Watts and S. H. Strogatz, *Nature* **393**, 440 (1998).
10. M. E. J. Newman and D. J. Watts, *Phys. Lett. A* **263**, 341 (2000).
11. C. Moore and M. E. J. Newman, Cond-mat 9911492 (2000).
12. D. Summers, J. G. Cranford, and B. P. Healy, *CSF* **11**, 2331 (2000).
13. M. Ding, C. Grebogi, and E. Ott, *Phys. Rev.* **39**, 2593 (1989).
14. J. D. Murray, *Mathematical Biology* (Springer-Verlag, New York, 1993).